PHD COURSE:
PARADIFFERENTIAL OPERATORS AND DYNAMICS OF NONLINEAR PDES

ROBERTO FEOLA

Duration. 20 h . **Exam:** Seminar

Description. We shall discuss several modern tools of micro-local analysis with application to the study of nonlinear partial differential equations. The aim of the course is to provide a self-contained introduction to para-differential operators and show how they can be used to prove a priori energy estimates and build up local existence theory for some type of quasi-linear partial differential equations. Time permitting, we shall discuss some applications to normal form theory for PDEs on compact manifolds.

Outline program. The course will be essentially divided into three parts. At first instance we will present some basic tools in harmonic analysis and we will provide an introduction to pseudo-differential symbols to discuss symbolic calculus: compositions, adjoints, quantizations. Then we will study the action of pseudo-differential operators in Sobolev spaces and generated flows.

In the second part of the course we shall introduce para-differential operators *via* quantizations of symbols with limited regularity. We will then prove the *paralinearizations theorems* to rewrite nonlinear expressions by para-differential expressions.

We will conclude with some applications to the Cauchy theory for some type of quasi-linear equations.

REFERENCES

[1] R. Feola and F. Iandoli. Local well-posedness for quasi-linear NLS with large Cauchy data on the circle. *Annales de l'Institut Henri Poincaré (C) Analyse non linéaire*, 36(1):119–164, 2018.

[2] R. Feola and F. Iandoli. Local well-posedness for the quasi-linear Hamiltonian Schrödinger equation on tori. *Journal de Mathématiques Pures et Appliquées*, 157:243–281, 2022.

[3] R. Feola, B. Grébert, and F. Iandoli. Long time solutions for quasilinear Hamiltonian perturbations of Schrödinger and Klein-Gordon equations on Tori. to appear on Analysis and PDEs, arXiv:2009.07553, 2020.

[4] R. Feola and F. Iandoli. Long time existence for fully nonlinear NLS with small Cauchy data on the circle. *Ann. Sc. Norm. Super. Pisa Cl. Sci.*, XXII(5):109–182, 2021.

[5] L. Hörmander, *The analysis of Linear partial differential operators III: Pseudo Differential Operators*, Springer 1994

[6] X. Saint Raymond, *Elementary Introduction to the Theory of Pseudodifferential Operators*, Studies in Advanced Mathematics, CRC Press, Boca Raton, 1991.

[7] E. Stein, *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*, Princeton University Press 1993.

[8] M.E. Taylor, *Pseudodifferential Operators And Nonlinear PDE*, Birkhäuser 1991.

Lecture Notes:

[9] G. Metivier, *Para-differential Calculus and Applications to the Cauchy Problem for Nonlinear Systems*

[10] R. Melrose, *Introduction to Microlocal Analysis*.